PUSAT SAINS MATEMATIK
FINAL EXAM ANSWER SHEET
SEMESTER: 1 SESSION: 2021/2022

Course : APPLIED STATISTICS
Course Code : BUM2413

QUESTION 1 (7 Marks)

$H_0 : \sigma^2 \geq (6400)^2$

$H_1 : \sigma^2 < (6400)^2$

$$\chi^2_{test} = \frac{19(5010)^2}{(6400)^2} = 11.6431$$

$\chi^2_{0.90,\ 19} = 11.6509$

Since $\chi^2_{test} = 11.6431 < \chi^2_{0.90,\ 19} = 11.6509$, Reject H_0

At $\alpha = 0.10$, there is enough evidence to support the claim that the addition of the new polymer in the new tire produces a more consistent tire mileage.

QUESTION 2 (13 Marks)

i)

$H_0 : \sigma_C^2 = \sigma_U^2$

$H_1 : \sigma_C^2 \neq \sigma_U^2$

$f_{0.02,7,7} = 5.4355$

$$\left(\frac{6.5257}{15.2193}\left(\frac{1}{5.4355}\right), \frac{6.5257}{15.2193}(5.4355)\right) = (0.0789, 2.3206)$$

Since $\sigma_0^2 = 1$ is within the interval $(0.0789, 2.3206)$, Do not reject H_0

At $\alpha = 0.04$, there is enough evidence to support the claim that both carpeted and uncarpeted rooms have equal population variances.

ii)

$H_0 : \mu_C - \mu_U \leq 0$
$H_1 : \mu_C - \mu_U > 0$

$P\text{-}value = 0.0668$

Since $\left(P\text{-}value = 0.0668\right) > \left(\alpha = 0.04\right)$, do not reject H_0

At $\alpha = 0.04$, there is enough evidence to reject the claim that the carpeted rooms are more likely to accumulate more bacteria compared to uncarpeted rooms.

QUESTION 3 (15 Marks)

i)

No of treatment = 12
List of treatment:
(Honda, A), (Honda, B), (Honda, C), (Honda, D)
(Toyota, A), (Toyota, B), (Toyota, C), (Toyota, D)
(Mazda, A), (Mazda, B), (Mazda, C), (Mazda, D)

ii) No. of replication = 2

iii)

$W = SSE$
$= \text{SST} - SSA + SSB + SSAB$
$= 0.00041$

$X = ab(r-1)$
$= 4(3)(2-1)$
$= 12$

$Y = F_{test}$
$= \dfrac{0.000031}{0.000034} = 0.9118$

$Z = f_{0.05,6,12} = 2.9961$

iv)

H_0 : There is no interaction effect on rolling friction coefficient of car speed between the four different types of engine oil and the three different brands of car

H_1 : There is an interaction effect on rolling friction coefficient of car speed between the four different types of engine oil and the three different brands of car

$\left(f_{test} = 0.9118\right) < \left(f_{0.05,6,12} = 2.9961\right)$, then we do not reject H_0

At $\alpha = 0.05$, there is no interaction effect between the four different types of engine oil and the three different brands of car.

v) Yes, because there is no interaction effect between the four different types of engine oil and the three different brands of car

QUESTION 4 (20 Marks)

i) Recovery period

ii) $r = \dfrac{1342.0769}{\sqrt{71676.7692 \times 70.3077}} = 0.5978$

There is a moderate positive correlation between the two variables

iii) $\bar{x} = 115.3077, \quad \bar{y} = 10.2308$

$\hat{\beta}_1 = \dfrac{1342.0769}{71676.7692} = 0.0187,$

$\hat{\beta}_0 = 10.2308 - 0.0187 \times 115.3077 = 8.0745$

$\hat{y} = 8.0745 + 0.0187x$

iv) $\hat{y} = 8.0745 + 0.0187(200) = 11.8145 \sim 12$ days

v)

Source of Variations	Sum of Squares	Degrees of Freedom	Mean of Squares	f_{test}
Regression	25.0968	1	**Q = 25.0968**	**S = 6.1062**
Residual	**P = 45.2109**	11	**R = 4.1100**	
Total	70.3077	12		

vi)

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$f_{test} = 6.1062$

$f_{0.04,1,11} = 5.4202$

Since $\left(f_{test} = 6.1062\right) > \left(f_{0.04,1,11} = 5.4202\right)$, we reject H_0.

At $\alpha = 0.04$, there is a linear relationship between the two variables.

QUESTION 5 (20 Marks)

i) Adjusted $r^2 = 0.5674$

56.74% of the variation in weekly pie sales can be predicted by the unit price, investment in advertising and the number of good reviews they received.

ii)

$H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

$H_1 : \beta_j \neq 0$ for at least one $j = 1, 2, 3$

$P - value = 0.0063$

Since $(P - value = 0.0063) < (\alpha = 0.08)$, we reject H_0

At $\alpha = 0.08$, at least one of the independent variables is related to the dependent variable.

iii) $\hat{y} = 308.6138 - 20.5602P + 47.5406A + 11.2042G$

iv)

When the unit price and number of good reviews are held constant, the estimated Pie sales increased by 47.5406 for an increase of RM100 in the investment of advertising.

v) Unit price: $(P - value = 0.0589) < (\alpha = 0.08)$, Price is significant

Investment in advertising: $(P - value = 0.0956) > (\alpha = 0.08)$, Advertising is not significant

Number of good reviews: $(P - value = 0.0578) < (\alpha = 0.08)$, No of good reviews is significant

Investment in advertising should not be considered

vi)

Predictor(s)	P-value	r^2	Adjusted r^2	Regression model
P	0.0979	0.1965	0.1347	$\hat{y} = 558.2772 - 24.0339P$
$\boldsymbol{A}$	**0.0313**	**0.3095**	**0.2564**	$\hat{y} = 147.6994 + 72.3086A$
G	0.0064	0.4473	0.4048	$\hat{y} = 304.3631 + 17.3726G$
P, A	**0.0118**	**0.5230**	**0.4435**	$\hat{y} = 184.2708 + 40.4603A + 13.5843G$
A, G	0.0118	0.5230	0.4435	$\hat{y} = 184.2708 + 40.4603A + 13.5843G$
P, G	0.0075	0.5574	0.4836	$\hat{y} = 433.5092 - 18.2670P + 15.8470G$
P, A, G	**0.0063**	**0.6601**	**0.5674**	$\hat{y} = 308.6138 - 20.5602P + 47.5406A + 11.2042G$

Best model: $\hat{y} = 304.3631 + 17.3726G$

Because it has $(P - value = 0.0064) < (\alpha = 0.08)$ and highest $r^2 = 0.4473$

vii)

$\hat{y} = 304.3631 + 17.3726(15)$

$= 564.9521 \sim 565$ units

QUESTION 6 (12 Marks)

H_0 : The number of mistakes per manuscript in the journal follows the Poisson distribution
H_1 : The number of mistakes per manuscript in the journal does not follows the Poisson distribution

X_i	O_i		P_i	E_i	
0	211		$P_1 = \frac{e^{-0.4}(0.4)^0}{0!} = 0.6703$	$E_1 = 0.6703(325) = 217.8475$	
1	90		$P_2 = \frac{e^{-0.4}(0.4)^1}{1!} = 0.2681$	$E_2 = 0.2681(325) = 87.1325$	
2	19	24	$P_3 = \frac{e^{-0.4}(0.4)^2}{2!} = 0.0536$	$E_3 = 0.0536(325) = 17.42$	20.02
3	5		$P_4 = \frac{e^{-0.4}(0.4)^3}{3!} = 0.0072$	$E_4 = 0.0072(325) = 2.34$	
4 or more	0		$P_5 = 1-(P_1+P_2+P_3+P_4)$ = 0.0008	$E_5 = 0.0008(325) = 0.26$	

$$\chi^2_{test} = \frac{(211-217.8475)^2}{217.8475} + \frac{(90-87.1325)^2}{87.1325} + \frac{(24-20.02)^2}{20.02}$$
$= 1.0784$

$$\chi^2_{0.01,(3-1)} = \chi^2_{0.01,2} = 9.2103$$

Since $(\chi^2_{test} = 1.0784) < (\chi^2_{0.01,2} = 9.2103)$, then we do not reject H_0

At $\alpha = 0.01$, there is enough evidence to conclude that the number of mistakes per manuscript in the journal follows a Poisson distribution.

QUESTION 7 (13 Marks)

i)

Types of vaccine	**Blood type**			
	A	**B**	**AB**	**O**
Sinovac	50	**30**	50	20
AstraZeneca	30	20	20	**30**

ii)

H_0 : The different blood types who had side effect do not relate to the types of vaccine.

H_1 : The different blood types who had side effect relate to the types of vaccine. (claim)

O_{ij}	$E_{ij} = \frac{n_{i.} \times n_{.j}}{n_{..}}$	$\frac{(O_{ij} - E_{ij})^2}{E_{ij}}$
$O_{11} = 50$	$E_{11} = \frac{150 \times 80}{250} = 48$	$\frac{(50-48)^2}{48} = 0.0833$
$O_{12} = 30$	$E_{12} = \frac{150 \times 50}{250} = 30$	$\frac{(30-30)^2}{30} = 0$
$O_{13} = 50$	$E_{13} = \frac{150 \times 70}{250} = 42$	$\frac{(50-42)^2}{42} = 1.5238$
$O_{14} = 20$	$E_{14} = \frac{150 \times 50}{250} = 30$	$\frac{(20-30)^2}{30} = 3.3333$
$O_{21} = 30$	$E_{21} = \frac{100 \times 80}{250} = 32$	$\frac{(30-32)^2}{32} = 0.125$
$O_{22} = 20$	$E_{22} = \frac{100 \times 50}{250} = 20$	$\frac{(20-20)^2}{20} = 0$
$O_{23} = 20$	$E_{23} = \frac{100 \times 70}{250} = 28$	$\frac{(20-28)^2}{28} = 2.2857$
$O_{24} = 30$	$E_{24} = \frac{100 \times 50}{250} = 20$	$\frac{(30-20)^2}{20} = 5$
		$\chi^2_{test} = 12.3512$

$\chi^2_{0.025,(4-1)(2-1)} = \chi^2_{0.025,3} = 9.3484$

Since $\left(\chi^2_{test} = 12.3512\right) > \left(\chi^2_{0.025,3} = 9.3484\right)$, then we reject H_0

At $\alpha = 0.025$, there is enough evidence to say that different blood type who had side effect relate to the type of vaccine.